# 1 3rd Year Design and Production

# Fatigue – Lecture 3

## 2 Fatigue

### 2.1 Revision

Fatigue strength as obtained from R. R. Moore fatigue tests can be used to predict the fatigue strength of other components. We use modifying factors:

$$S_n = S_n' C_L C_G C_S$$

- $S_n$ is the actual (calculated) fatigue strength
- $S_n'$ is data from SN curve (experiment or estimate)
- $C_L$ modifying factor for **loading type**
- $C_D$ (a.k.a. $C_G$) modifying factor for **gradient/size**
- $C_S$ modifying factor for **surface finish**

In general $S_n < S_n'$

# 3 Endurance Limit Factors

| | Bending | Axial | Torsion |
|---|---|---|---|
| **(a) Endurance Limit** $S_n = S_n' C_L C_G C_S$ | | | |
| $C_L$ – Load Factor | 1 | 1 | 0.58 |
| $C_S$ – Surface Factor | chart | chart | chart |
| $C_D$ – Dimension Factor<br>$(0 < \varnothing < 10\text{mm})$ | 1.0 | 0.7 to 0.9 | 1.0 |
| $C_D$ – Dimension Factor<br>$(10\text{mm} < \varnothing < 50\text{mm})$ | 0.9 | 0.7 to 0.9 | 0.9 |
| **(b)** $10^3$ **Cycle Strength** | $0.9\ S_u$ | $0.75\ S_u$ | $0.9\ S_{us}$ |

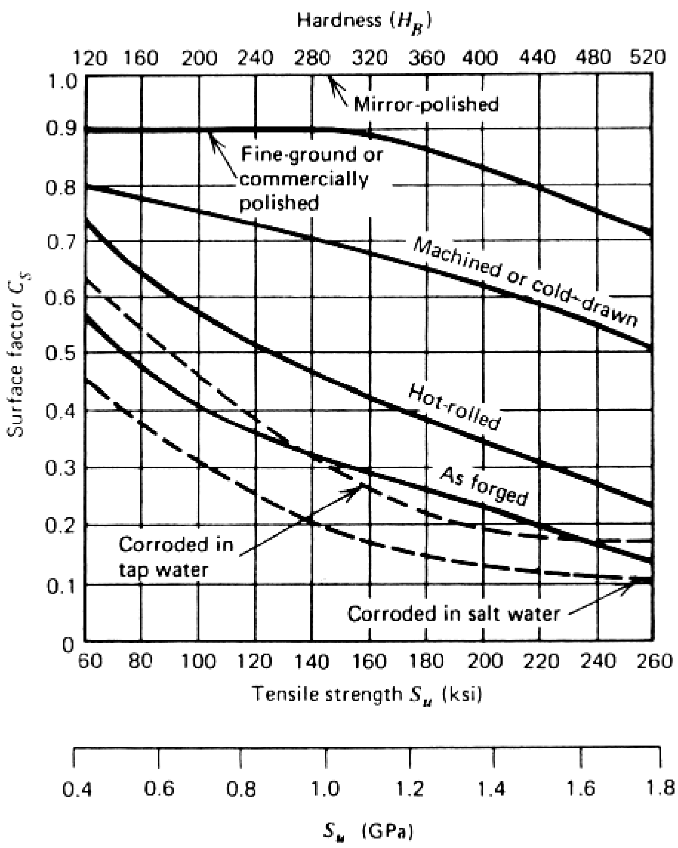
$$S_n' \approx 0.5\ S_u$$
$$S_u \approx 0.5(Bhn)[ksi]$$
$S_{us} \approx 0.8\ S_u$ for steel

$S_{ys} \approx 0.58\ S_y$ Max distortion energy theory

$S_{ys} \approx 0.50\ S_y$ Max shear stress theory

# 4 Endurance Limit Factors

## 4.1 Surface Finish



# 5 Mean and Alternating Stress

- We have been discussing **Fully Reversed Stress States** $\Rightarrow$ mean stress is zero.
- In many real situations, this is not the case.
- Some notation:
  - ◊ $\sigma_m$ mean stress
  - ◊ $\sigma_{max}$ maximum stress
  - ◊ $\sigma_{min}$ minimum stress
  - ◊ $\sigma_a$ amplitude of fluctuating component
- A Mathematical example:
  - ◊ $\sigma = \sigma_m + \sigma_a \sin(\omega t)$
  - ◊ $\Rightarrow \sigma_{max} = \sigma_m + \sigma_a$ and $\sigma_{min} = \sigma_m - \sigma_a$
  - ◊ $\Rightarrow \sigma_m = 0.5(\sigma_{max} + \sigma_{min})$

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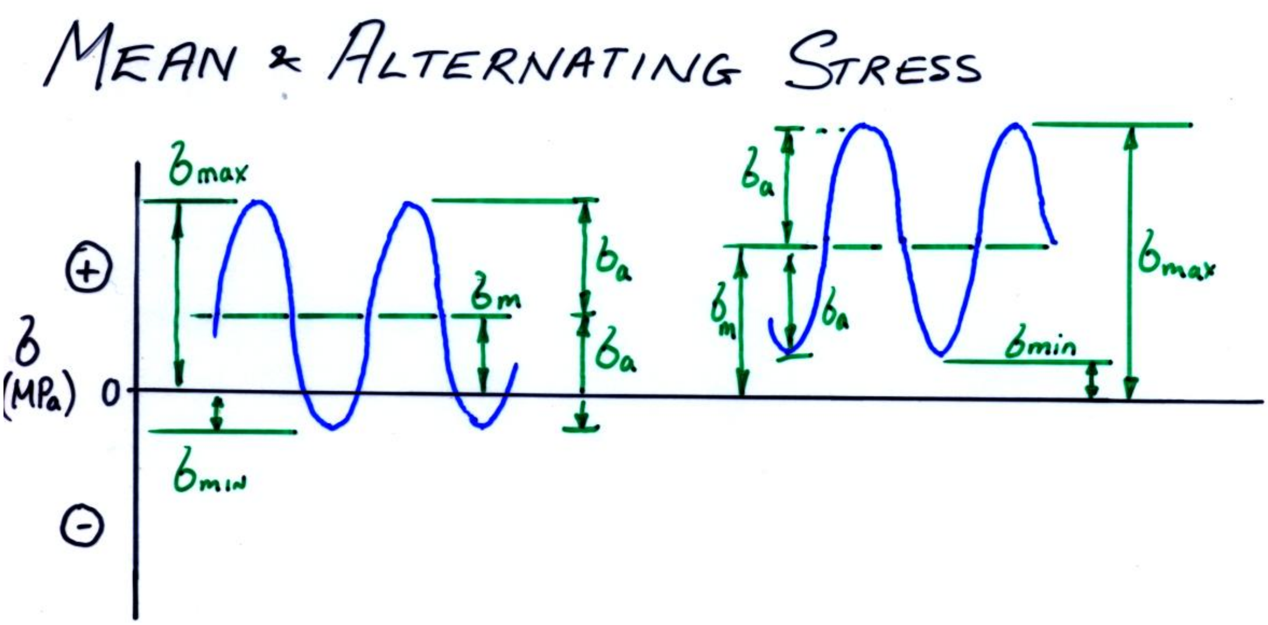
# 6 Mean and Fluctuating Stress




- Presence of a Static **Tensile** Stress reduces the amplitude of Reversed Stress that can be applied
- Presence of a Static **Compressive** Stress slightly increases amplitude of Reversed Stress that can be applied (effect usually ignored)

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# 7 Constant Life Fatigue Diagrams

- To look at effect/interaction of $\sigma_m$ and $\sigma_a$, we aim to draw a constant life fatigue curve.
- x–axis: mean stress $\sigma_m$
- y–axis: fluctuating amplitude $\sigma_a$
- If $\sigma_a = 0$, **Static Failure** (intersection of curve with x–axis)
  - May use $S_y$ or $S_u$ depending on application
- If $\sigma_m = 0$, **Totally Reversed Load** (intersection of curve with y–axis)
  - Data (maybe modified) from $SN$ diagram: $S_n$ (for infinite life)

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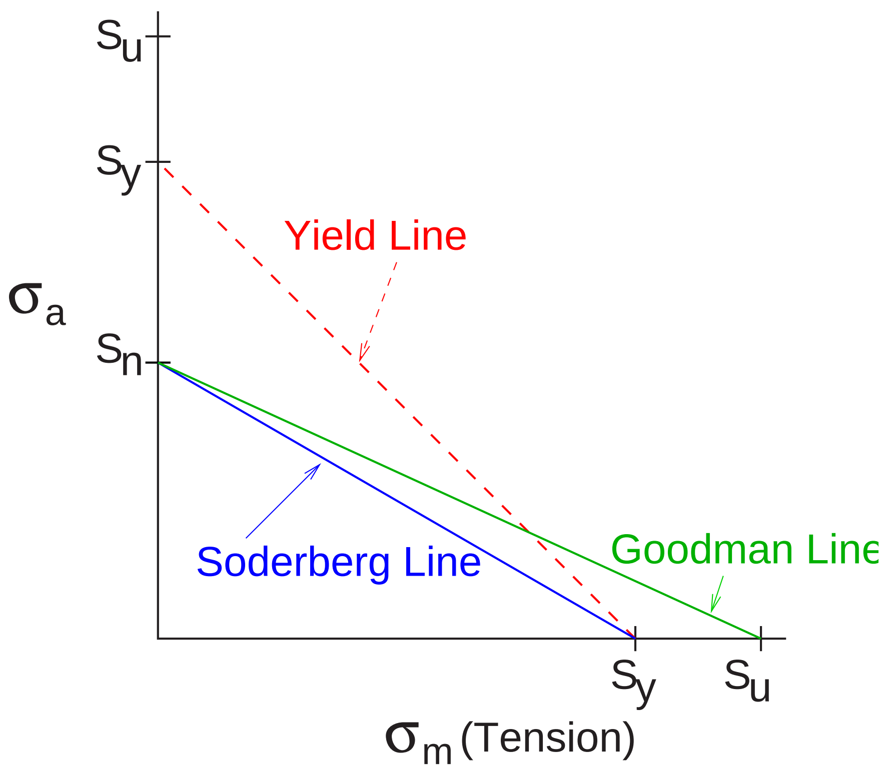
# 8 Constant Life Fatigue Diagrams

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# 9 Constant Life Fatigue Diagrams

## 9.1 Equations

- Soderberg Line is based on $S_y$ and $S_n$

  $\rightarrow$ $S_a/S_n + S_m/S_y = 1$

- Goodman Line is based on $S_u$ and $S_n$

  $\rightarrow$ $S_a/S_n + S_m/S_u = 1$

- Gerber Locus is based on $S_u$ and $S_n$

  $\rightarrow$ $S_a/S_n + (S_m/S_u)^2 = 1$
  $\rightarrow$ Parabolic

- Distortion Energy Locus is based on $S_u$ and $S_n$

  $\rightarrow$ $(S_a/S_n)^2 + (S_m/S_u)^2 = 1$
  $\rightarrow$ Elliptical

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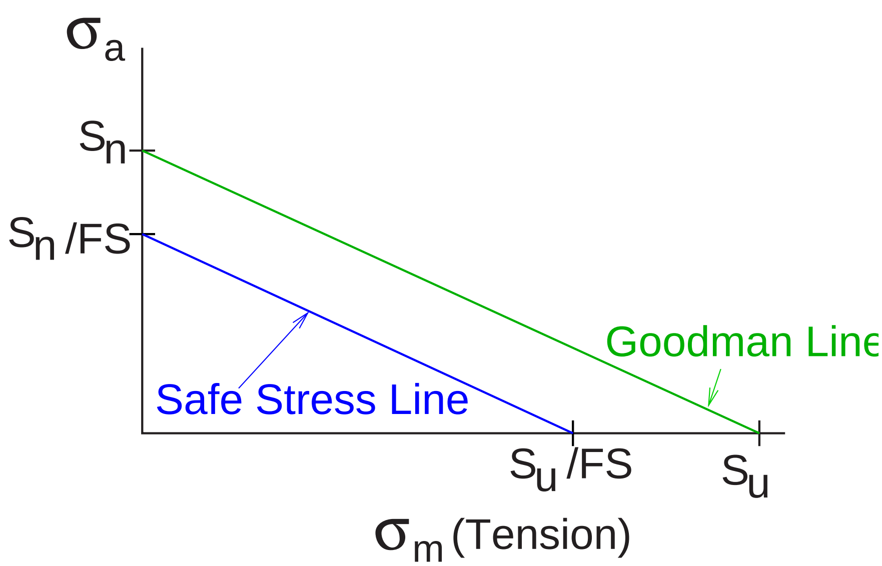
## 10 Factor of Safety

- Illustrated here for Goodman Criteria (can use Soderberg, etc., also).
- $FS$ is the factor of safety
- Safe stress line is parallel to original Goodman line

$$\frac{S_a}{S_n} + \frac{S_m}{S_u} = \frac{1}{FS}$$

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## 11 Summary of CLF Diagrams

- Four lines relevant for infinite life:
  - Soderberg, Goodman, Yield-line, Load-line

- Two additional failure loci (less conservative)
  - Gerber (parabolic), Distortion-Energy (elliptical)
- Which failure criteria to use?:
  - If no yielding allowed, Soderberg will be conservative
  - If we don't know $S_u$, then use Soderberg
  - If $S_u$ **is** known, and we want to find infinite life, with yielding permitted, then use Goodman line
  - If $S_u$ **is** known, and we want to find infinite life, with yield onset permitted, then use Goodman line and yield line to define the safe zone (Modified Goodman Criterion).

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Two acetates

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# 12 Stress Concentration

- Surface features and flaws can lead to locally elevated stresses
- Tables/charts allow us to look up $K_t$
  - Theoretical or Geometric stress concentration factors
- Materials have different **notch sensitivities**: $q$
  - Material with a lot of flaws not damaged much by one more
    $\Rightarrow$ low notch sensitivity
  - Very perfect material is significantly damaged by addition of a notch
    $\Rightarrow$ high notch sensitivity
- Calculate **Fatigue Stress Concentration Factor** $K_f$ using $K_t$ and $q$:

$$K_f = 1 + q(K_t - 1)$$

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# 13 Stress Concentration

What do we apply $K_f$ to?

- Ductile Materials
  - Nominal Mean Stress Method
  - Apply $K_f$ only to alternating stress
- Brittle Materials
  - Residual Stress Method
  - Apply $K_f$ to both alternating stress and mean stress