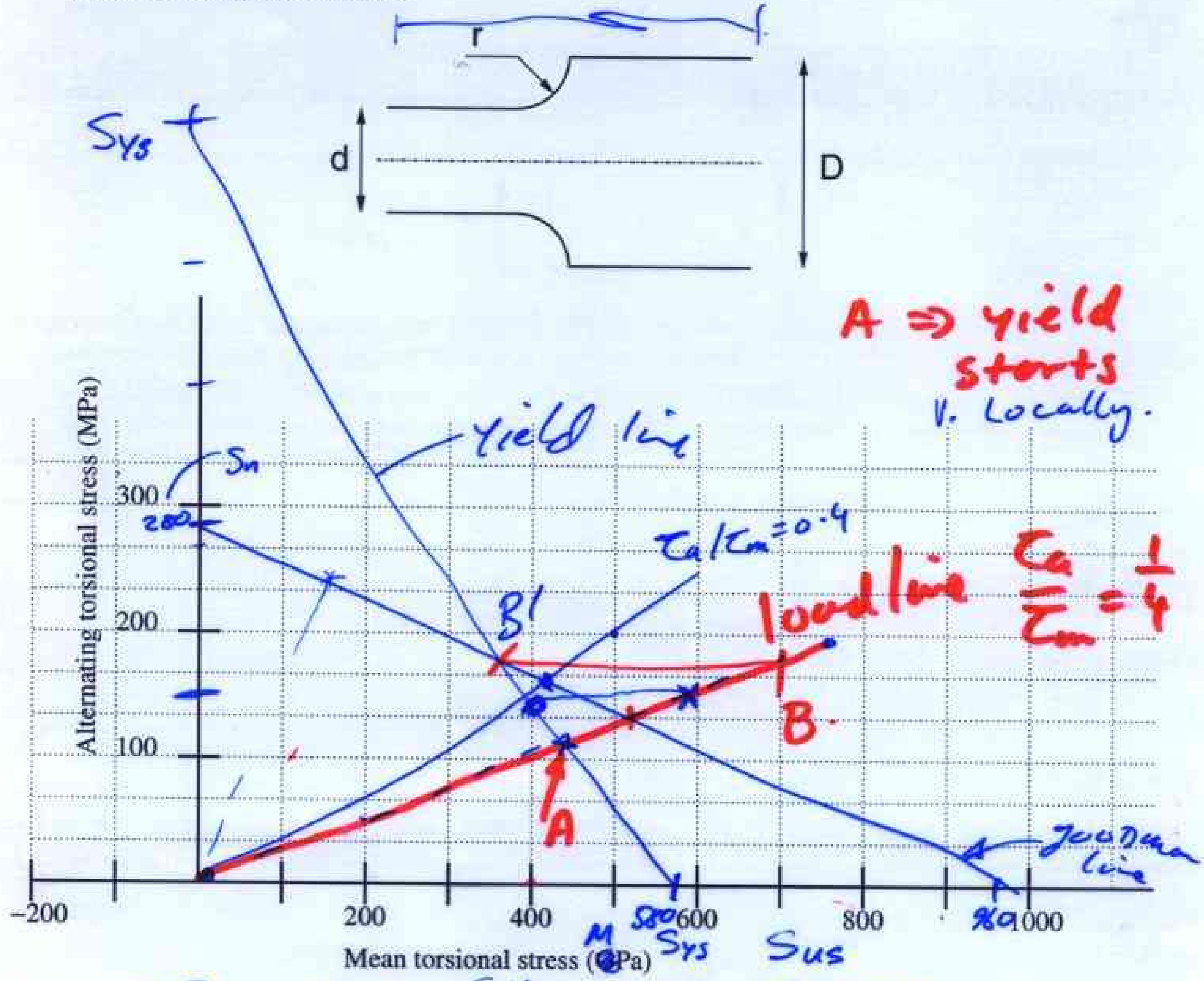
9th-January-2004

Problem A shaft must transmit a torque of 1000 Nm, with superimposed torsional vibrations causing an alternating torque of 250 Nm. A safety factor of 2 should be applied to both loads.

A heat treated alloy steel is to be used, with $S_u = 1.2$GPa and $S_y = 1.0$GPa. The shaft will have a shoulder, with D/d=1.2 and r/d=0.05 as show in the diagram. A good quality commercial ground finish is to be specified, what diameter is required for infinite life?



$S_n = ?$ $\quad S_n' \approx \frac{S_u}{2} = \frac{1200}{2} = 600\text{ MPa}$

$S_n = C_L C_D C_S S_n$

Torsion $\Rightarrow C_L = 0.58$; $C_S = 0.9$ (ground).

C_D depends on d assume $10 < \varnothing < 50$ mm

$C_D = 0.9$ (for TORSION).

$S_n = (600)(0.58)(0.9)(0.9) \approx 280\text{ MPa}$

$S_{us} \approx (0.8)(S_u) = 960\text{ MPa}$ $\quad S_{ys} \approx (0.58)\, S_u = 580\text{ MPa}$

9th - Jan 2004

$\tau_m = ?$ $\quad \tau_a = ?$

$$\tau_m = \left(\frac{16\,T_m}{\pi d^3}\right) \underline{k_f} \qquad \tau_a = \left(\frac{16\,T_a}{\pi d^3}\right) \underline{k_f}$$

need k_f → need K_t & q

K_t $\quad r/d = 0.05 \quad D/d = 1.2$

$K_t = 1.57$ from Handout.

q depends on r, which we don't know

⇒ be conservative

just use S_u to get q

⇒ $q = 0.95$

$$K_f = 1 + (1.57 - 1)(0.95) \simeq 1.54$$

subst into τ_m τ_a

$$\tau_m = \overset{\text{F.S.}}{2}\left[\frac{16\,(1000)}{\pi d^3}\right]\underset{1.54}{\overset{k_f}{1}} = 15685/d^3$$

$$\tau_a = 2\left[\frac{16\,(250)}{\pi d^3}\right]\underset{k_f}{\overset{1.54}{1}} = \frac{3922}{d^3}$$

$$\left[\frac{\tau_a}{\tau_m} = 0.25 \text{ approx.}\right.$$ ← load line

9th-Jan-2004

If you apply K_f <u>only</u> to

~~the~~ τ_a

$$\tau_a = \frac{3922}{d^3} \qquad \tau_m = \frac{15685}{1.54\,d^3}$$

$$\approx \frac{10000}{d^3}$$

$$\frac{\tau_a}{\tau_m} = 0.4 \quad (\text{not } 0.25)$$

this load line has also been plotted on the constant Life fatigue diagram, and it will give us a somewhat different failure point

either way, find τ_a at failure (approx 150 MPa)

Substitute into

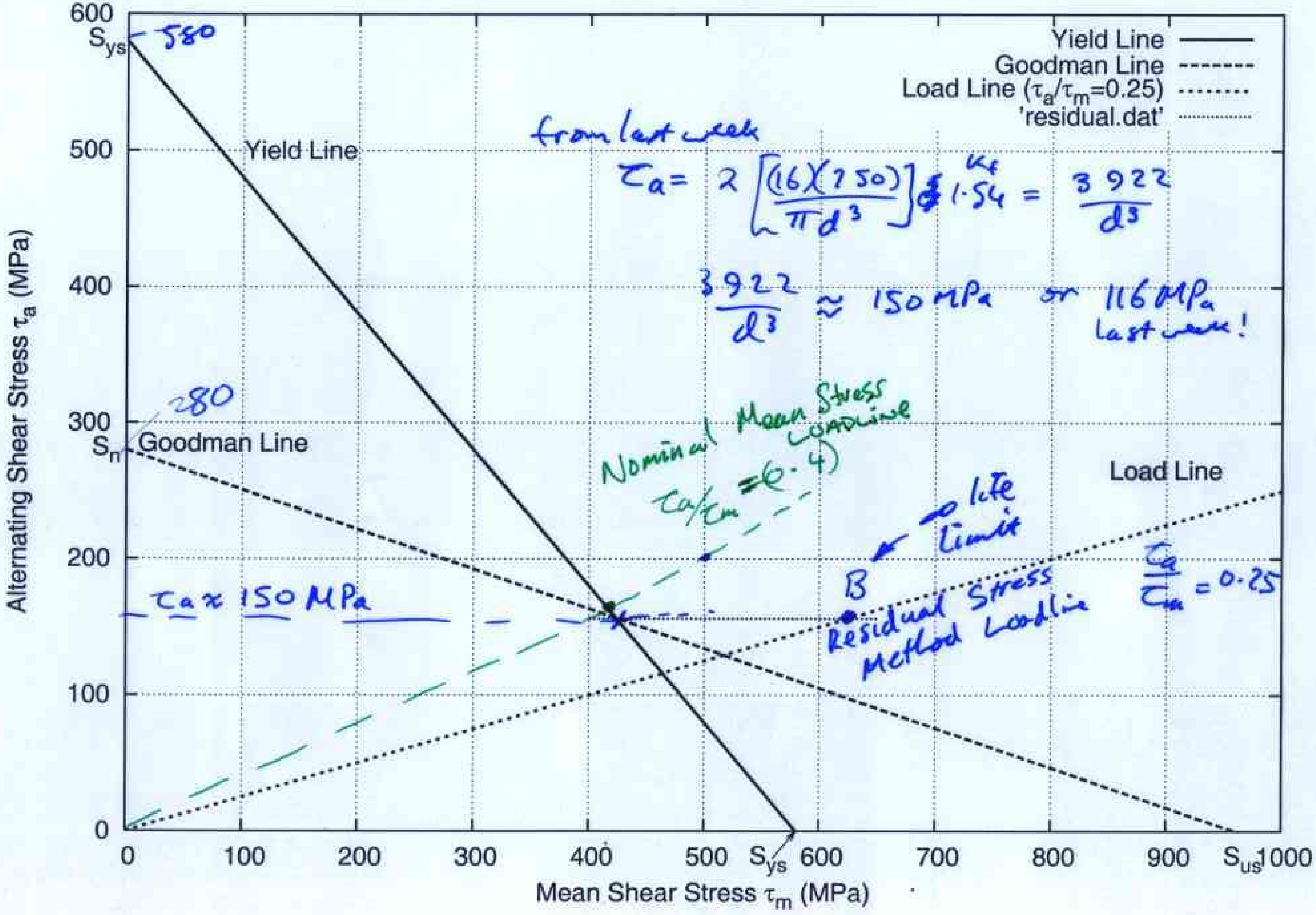
$$\tau_a = \frac{3922}{d^3} \Rightarrow d^3 = \frac{3922}{\tau_a}$$

need to be careful with units.

τ_a was calculated in terms of Nm, so stick with that

$$\tau_a = 150\text{ MPa} = 150 \times 10^6\text{ Pa}$$

$$d = \sqrt[3]{\frac{3922}{150 \times 10^6}} = 0.0296 \simeq 30\text{ mm}$$



Yield Line
Goodman Line
Load Line (τ_a/τ_m=0.25)
'residual.dat'
Alternating Shear Stress τ_a (MPa)
Mean Shear Stress τ_m (MPa)
S_{ys} 580
S_n 280
S_{ys}
S_{us}
Yield Line
Goodman Line
Load Line
from last week
$\tau_a = 2\left[\frac{(16)(750)}{\pi d^3}\right] \overset{K_f}{1.54} = \frac{3922}{d^3}$
$\frac{3922}{d^3} \approx 150$ MPa or 116 MPa last week!
Nominal Mean Stress LOADLINE
$\tau_a/\tau_m = 0.4$
$\tau_a \approx 150$ MPa
life limit
B
Residual Stress Method Loadline
$\frac{\tau_a}{\tau_m} = 0.25$