1 3rd Year Design and Production

Fatigue – Lecture 6

2 Cumulative Fatigue Damage

We have studied varying loads. However, we have assumed that σ_m and σ_a have not varied over time. Often this is not the case.

2.1 Palmgren/Miner Rule

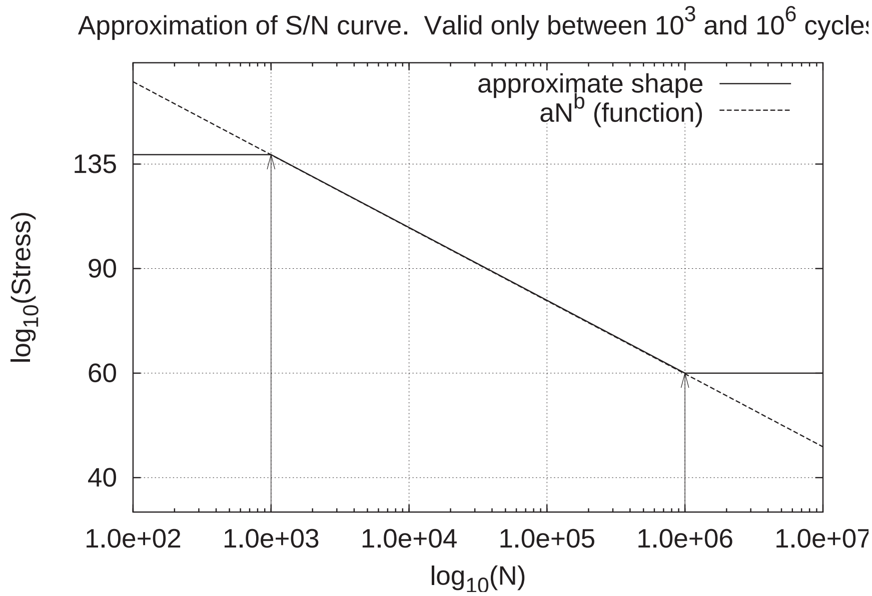
- If n_1, n_2, n_3, n_4, $\ldots$, n_k, are the number of cycles accumulated at specific stress levels
- And N_1, N_2, N_3, N_4, $\ldots$, N_k, are the lifetimes predicted at these stress levels
- Then failure will occur when

$$\sum_{j=1}^{j=k} \frac{n_j}{N_j} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} + \cdots + \frac{n_k}{N_k} = 1$$

3 Cumulative Fatigue Damage

3.1 Estimating life at a stress

How do we know what the life of the component is at a particular stress?
i.e. how do we get N_i for σ_i?
Recall, our $S - N$ curves related stress to lifetime (in cycles).



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4 Cumulative Fatigue Damage

4.1 Estimating Life at a Stress – SN Curve

We need two points on the log-log curve to (approximately) draw it.

- 10^3 cycle limit
- Endurance limit $\equiv 10^6$ cycle limit $\equiv S_n$

There are approximate expressions for estimating these in the handouts.
(Experimental data would be better, of course.)
For example

- $S_3 = 0.9S_u$, for bending
- $S_n = C_L C_S C_D S'_n = C_L C_S C_D (0.5S_u)$

With these two points, we can draw on log-log paper the SN curve, and then we can read off any intermediate lifetime (given a stress level)

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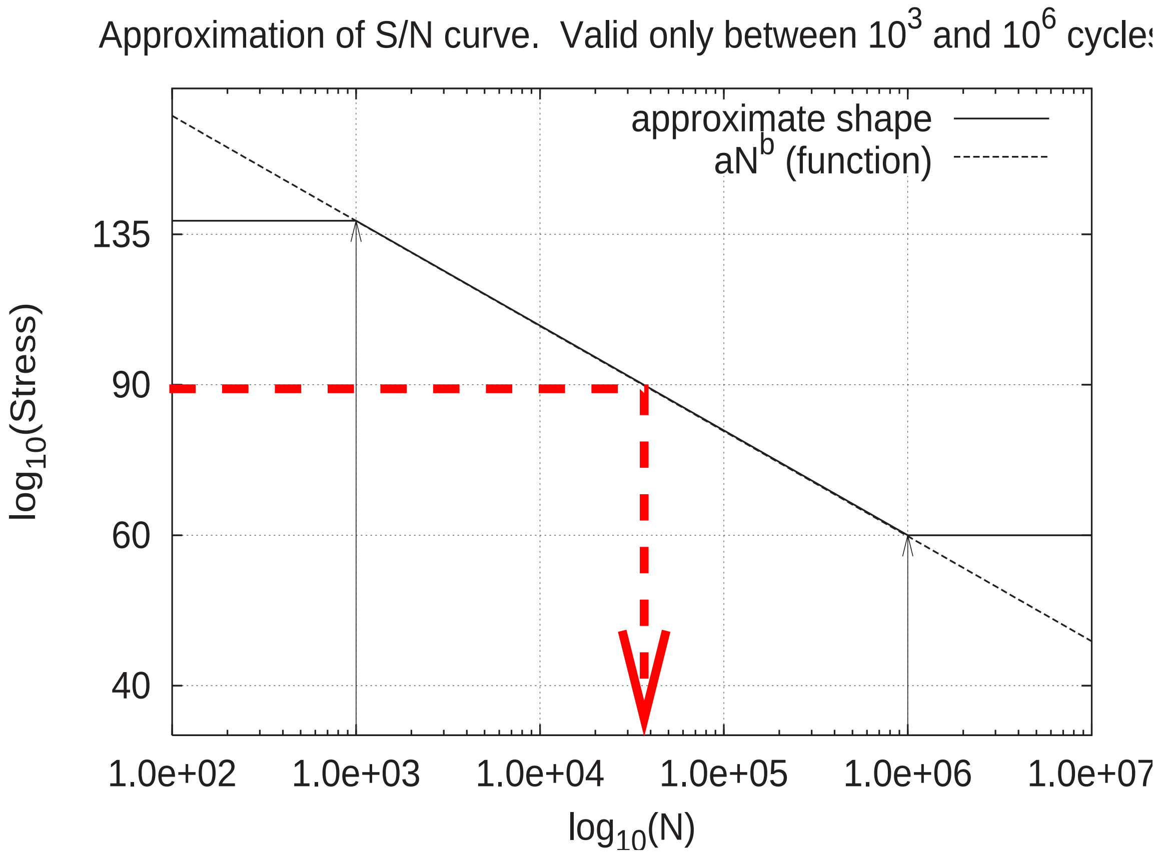
5 Cumulative Fatigue Damage



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6 Cumulative Fatigue Damage

6.1 Estimating life at a stress – Calculation

We can also calculate the intermediate points
Recall basic maths: the equation for a straight line:

$$y = mx + c$$

Since we have a straight line on our log-log plot, we can say

$$\log_{10}(S_f) = b\log_{10}(N) + \log_{10}(a) \qquad \text{or equivalently} \qquad S_f = aN^b$$

There are two unknowns, a and b. We can find these using our two known points at 10^3 and 10^6 cycles.

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7 Cumulative Fatigue Damage

7.1 Estimating life at a stress – Calculation

Say a material has a 10^3 cycles strength of 140ksi, and an endurance limit of 60ksi, then we can say

$$\log_{10}(140) = b\log_{10}(10^3) + \log_{10}(a) = 3b + \log_{10}(a)$$
$$\log_{10}(60) = b\log_{10}(10^6) + \log_{10}(a) = 6b + \log_{10}(a)$$

This gives two simple equations...

$$2.146 = 3b + \log_{10}(a)$$
$$1.778 = 6b + \log_{10}(a)$$

We can then solve to get b and $log_{10}(a)$.
Later, for any given S_f, we find N using...

$$N = 10^{\left(\log_{10}(S_f) - \log_{10} a\right)/b}$$

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8 Mean and Alternating Loads

What we have done so far is sufficient for fully reversed loading. . .
what about loads that have a mean component?

8.1 Constant Life Fatigue Diagram

We use the same tool we used when looking at infinite life, the CLF-diagram

- Goodman line
- Every point on Goodman (or Soderberg if that is preferred) line has the same lifetime
- Find the fully reversed stress with the same life as our mean plus alternating
- Find the lifetime for that equivalent alternating stress using the SN curve.

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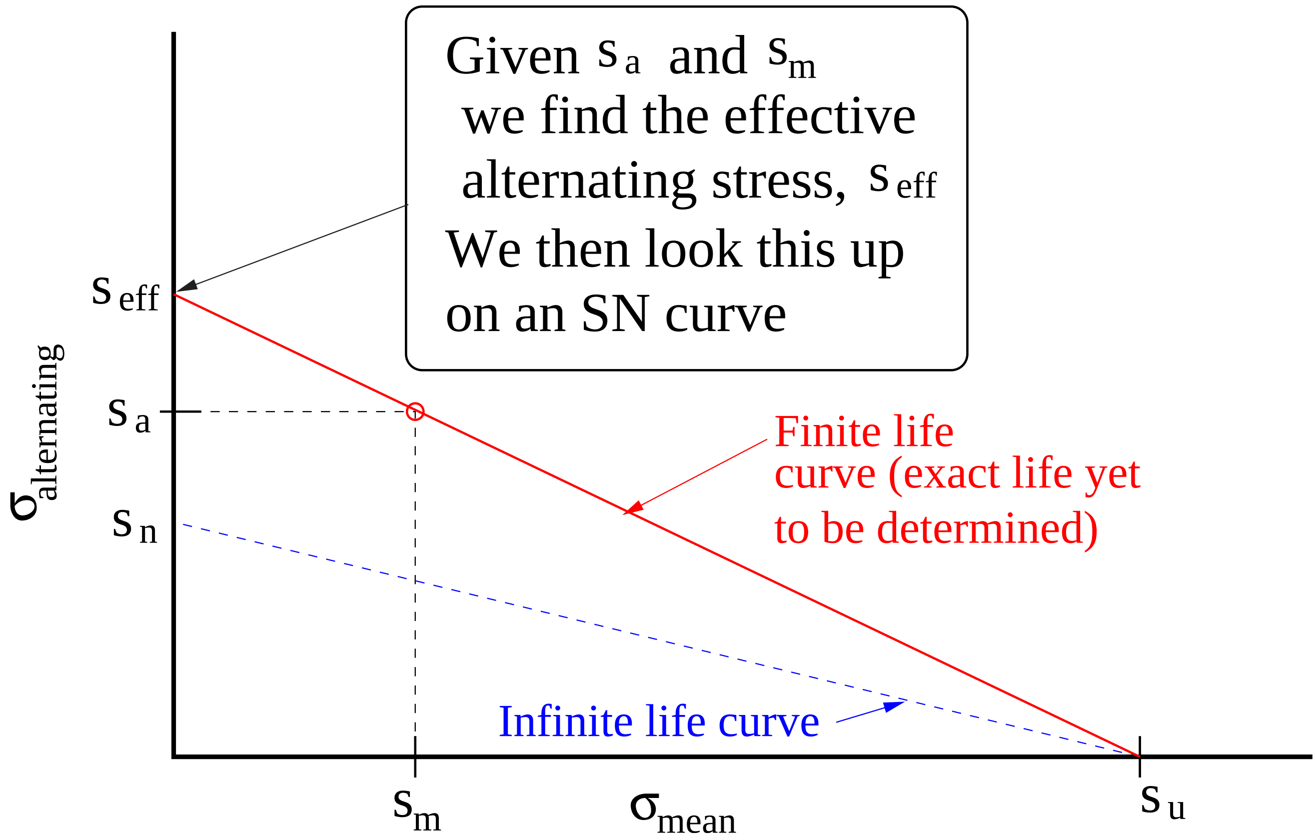
9 Mean and Alternating Loads



10 Sample Problem

A critical notch is subject to varying nonsteady loading. A typical 6 second period includes the following loading condition

- 2 cycles at $\sigma_a = 100\text{M}Pa$ and $\sigma_m = 50\text{M}Pa$
- 4 cycles at $\sigma_a = 125\text{M}Pa$ and $\sigma_m = 75\text{M}Pa$
- 2 cycles at $\sigma_a = 225\text{M}Pa$ and $\sigma_m = 125\text{M}Pa$
- 1 cycle at $\sigma_a = 350\text{M}Pa$ and $\sigma_m = 50\text{M}Pa$

The part is made from aluminium, and has the following properties: $S_u = 480$MPa, $S_y = 410$MPa. Correcting for geometry, surface, etc., the fatigue properties of the notch are: $S_{10^3} = 450$MPa, $S_{10^6} = 180$MPa.

Calculate the expected life of the component.