# KELVIN MODEL

$$\sigma = k\varepsilon + \mu\dot{\varepsilon}$$

**CREEP**: constant stress $\sigma_0$

$$\sigma_0 = k\varepsilon + \mu\dot{\varepsilon}$$ ← Differential EQN

Solution $$\varepsilon = \frac{\sigma_0}{k}\left[1 - e^{-\left(\frac{kt}{\mu}\right)}\right]$$

$t=0$ $e^{-\frac{k0}{\mu}} = 1 \Rightarrow \varepsilon = 0$

$t \to \infty$ $e^{-\frac{kt}{\mu}} \to 0$ $\varepsilon \Rightarrow \frac{\sigma_0}{k}$ strain you'd get with spring alone

$\varepsilon$ goes from $0$ to $\frac{\sigma_0}{k}$ asymptotically

**RELAXATION**; constant STRAIN $\Rightarrow \dot{\varepsilon} = 0$

$\dot{\varepsilon} = \frac{d\varepsilon}{dt}$ $\varepsilon = \varepsilon'$

$$\sigma = k\varepsilon' + \cancel{\mu(0)}^{\,0} \Rightarrow \sigma = k\varepsilon'$$

$\sigma$ is constant too.

**RECOVERY** REMOVE STRESS WHILE @ strain level $\varepsilon'$ $\sigma = 0$

$$0 = k\varepsilon + \mu\dot{\varepsilon}$$

$$\varepsilon = \varepsilon' e^{-\left[\frac{kt}{\mu}\right]}$$ i.e. another asymptote.

$[t=0, \varepsilon=\varepsilon']$ $[t\to\infty, \varepsilon\to 0]$

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MAXWELL MODEL:

IN SERIES

LD $\sigma = \sigma_1 = \sigma_2$

$\varepsilon = \varepsilon_1 + \varepsilon_2$

$\Rightarrow \dot{\varepsilon} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2$

$\sigma_1 = k\varepsilon_1$ spring

$\sigma_2 = \mu\dot{\varepsilon}_2$ DAMPER

$= \mu \frac{d\varepsilon_2}{dt}$

$\dot{\varepsilon}_2 = \frac{\sigma_2}{\mu}$

$\varepsilon_1 = \frac{\sigma_1}{k} \Rightarrow \dot{\varepsilon}_1 = \frac{\dot{\sigma}_1}{k}$

$$\boxed{\dot{\varepsilon} = \frac{\sigma_2}{\mu} + \frac{\dot{\sigma}_1}{k} = \frac{\sigma}{\mu} + \frac{\dot{\sigma}}{k}}$$

CREEP const STRESS $= \sigma_0$

$\Rightarrow \dot{\sigma} = 0$

$\dot{\varepsilon} = \frac{\sigma_0}{\mu} + 0 = \text{constant}.$

$\varepsilon$ increasing in a straight line FOREVER. with no limit

THERE <u>WILL</u> BE AN INITIAL ELASTIC STRETCH EQUAL TO $\varepsilon = \frac{\sigma_0}{k}$

i.e. EQUAL TO final limit of KELVIN MODEL.

# STRESS RELAXATION

↳ CONST STRAIN $\Rightarrow \dot{\varepsilon} = 0$

$$0 = \frac{\sigma}{\mu} + \frac{\dot{\sigma}}{k} \Rightarrow \text{DIFFERENTIAL EQUATION}$$

$$\sigma = \sigma_0 e^{-\left(\frac{kt}{\mu}\right)} \qquad \sigma = \sigma_0 \ @\ t = 0$$

# RECOVERY

↳ REMOVE STRESS

ELASTIC STRAIN RECOVERS INSTANTLY

$\Rightarrow \varepsilon_1 \to 0$

NO further recovery.

$\sigma = 0 \qquad \dot{\sigma} = 0$

$\dot{\varepsilon} = 0$

ELASTIC RECOVERY $= \dfrac{\sigma'}{k}$ ← STRESS IN SYSTEM at MOMENT of ~~RELAXATION~~ LOAD REMOVAL

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