1 4th Year Materials Engineering

Mechanics of Composite Materials – Lecture 5

2 Last Week

2.1 Summary

http://mconry.ucd.ie/%7emconry/4th_Materials_Engineering/

- Calculated composite properties based on components
  * $E_1 = E_f\phi_f + E_m(1-\phi_f)$
  * $E_2 = \frac{E_f E_m}{E_f\phi_m + E_m\phi_f}$
  * $\nu_{12} = \phi_f\nu_f + \phi_m\nu_m$
  * $\nu_{21}/E_2 = \nu_{12}/E_1$
- ϕ_f is the fibre volume fraction
- $\phi_m = 1-\phi_f$ is the matrix volume fraction

3 Failure of Unidirectional Composite Laminates

3.1 Tensile Stress Parallel to Fibres

Failure sequence depends on:

- Strength and Brittleness of Matrix
- Strength and Brittleness of Fibre
- Fibre Volume Fraction ϕ_f

Something is **Brittle** if it fails at small strain.
Something is **Ductile** if it deforms significantly before it fails
These are both relative terms

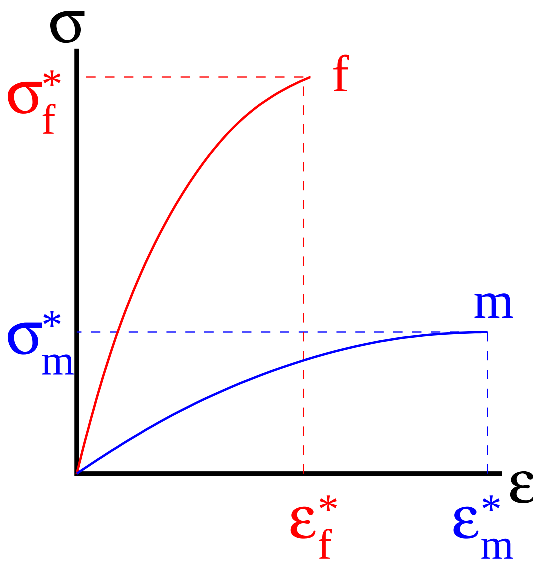
In other words, porcelain is brittle because it won't deform much before it fails, while rubber bands have very low brittleness. However, somethig can be brittle and still support a very high stress (if it is very stiff) . This is clear from the next figures.

4 Failure of Unidirectional Composite Laminates

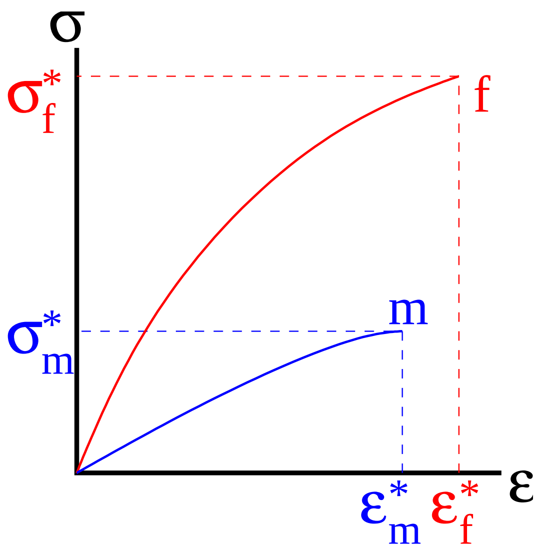
5 Failure of Unidirectional Composite Laminates

5.1 Brittle Fibres

E.g. Epoxy reinforced by carbon fibres. Assume that there is parallel coupling between the fibres and matrix, and also that stress applied is parallel to the fibres. Also, assume that the fibres and matrix



(a) Brittle Fibre, Ductile Matrix, e.g. Carbon fibre reinforced Epoxy



(b) Brittle Matrix, Ductile Fibre, e.g. Glass fibre reinforced thermoset-polyester

Figure 1: Fibre and Matrix Failure

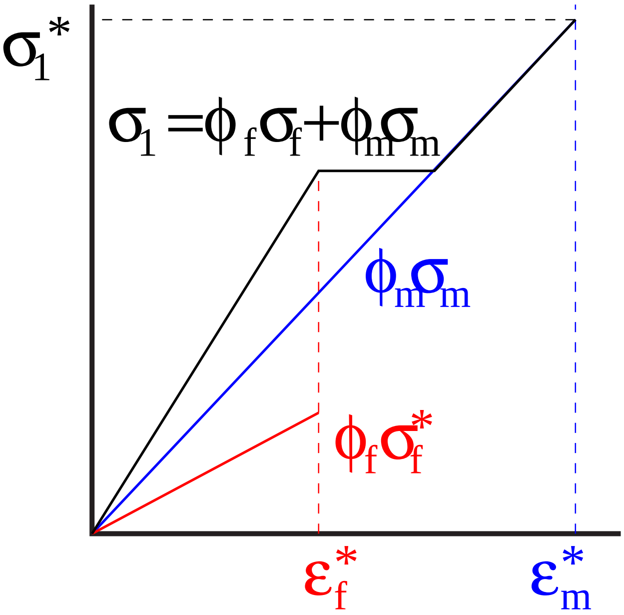
in the composite fail independently at the same stresses and strains as they would when tested in tensile tests of the pure material. For tensile loading parallel to fibres:

- Plot stress × volume-fraction (i.e. $\sigma\phi$) as a function of strain (ϵ)
- Superimpose curves for fibre & matrix
- Total stress = sum of $\phi_f\sigma_f$ and $\phi_m\sigma_m$
  - $\sigma_1 = \phi_f\sigma_f + \phi_m\sigma_m$
- At ϵ_1=ϵ_f^*, fibres fail
- Depending on ϕ_f, composite **might** fail, or might survive a while longer

6 Failure of Unidirectional Composite Laminates

6.1 Brittle Fibres: Low ϕ_f

- $\sigma_1 = \phi_f \sigma_f + (1 - \phi_f)\sigma_m$
- At $\epsilon_1 = \epsilon_f^*$, fibres fail
- Total Stress is transferred to matrix $\Rightarrow$ strain increases
- Stress can increase, until $\epsilon_1 = \epsilon_m^*$ is reached
  - Note stiffness is reduced
- Then Matrix fails
  $\Rightarrow$ Composite fails also



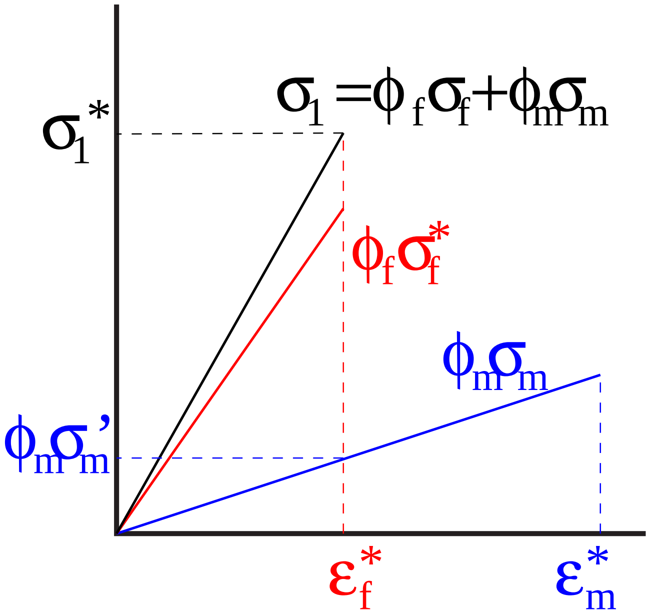
6.2 Failure Criterion I:

$$\sigma_1^* = (1 - \phi_f)\sigma_m^* \qquad \text{Low } \phi_f$$

7 Failure of Unidirectional Composite Laminates

7.1 Brittle Fibres: High ϕ_f

- $\sigma_1 = \phi_f \sigma_f + (1 - \phi_f)\sigma_m$
- At $\epsilon_1 = \epsilon_f^*$, fibres fail
- Total Stress is (potentially) transferred to matrix
- Matrix fails immediately as it cannot support the entire load
  $\Rightarrow$ Composite fails also



7.2 Failure Criterion II:

$$\sigma_1^* = \phi_f \sigma_f^* + (1-\phi_f)\sigma_m' \qquad \text{High } \phi_f$$

8 Failure of Unidirectional Composite Laminates

8.1 Brittle Fibres: Failure Criteria

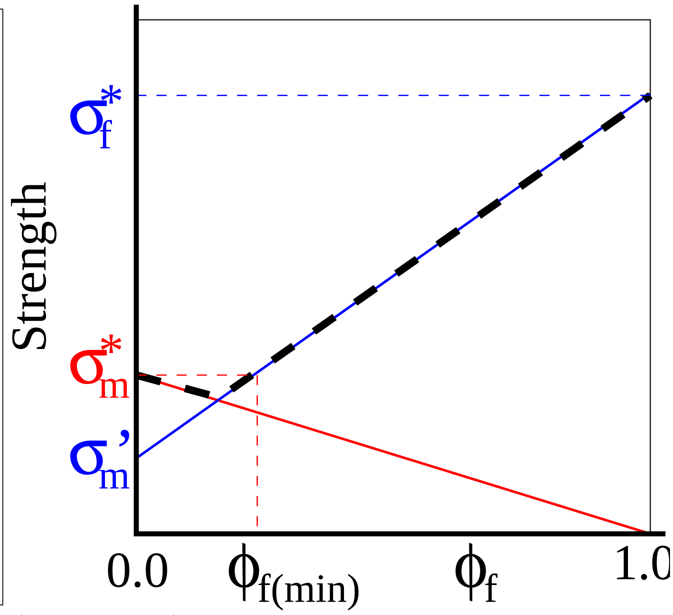
$$\sigma_1^* = (1-\phi_f)\sigma_m^* \qquad \text{Low } \phi_f$$
$$\sigma_1^* = \phi_f \sigma_f^* + (1-\phi_f)\sigma_m' \qquad \text{Big } \phi_f$$

Note: $\sigma_m' = \sigma_m(\epsilon_f^*)$

- Use whichever is higher (heavy dashed line in Figure).
- Note v. small ϕ_f **weakens** matrix! **But it will still stiffen the matrix somewhat.**

$$\phi_{f_{\min}} = \frac{\sigma_m^* - \sigma_m'}{\sigma_f^* - \sigma_m'}$$

- For carbon-fibre reinforced epoxy $\phi_{f_{\min}} = 0.03$.



9 Failure of Unidirectional Composite Laminates

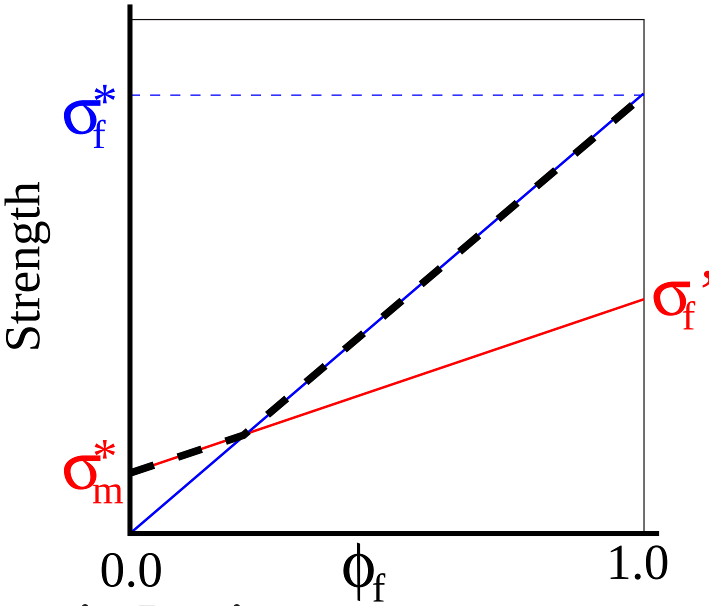
9.1 Ductile Fibres: Failure Criteria

Follow broadly simiar reasoning to brittle fibre case. Essentially switch roles of fibre and matrix in arguments

Example: Glass fibre reinforced thermoset polyester

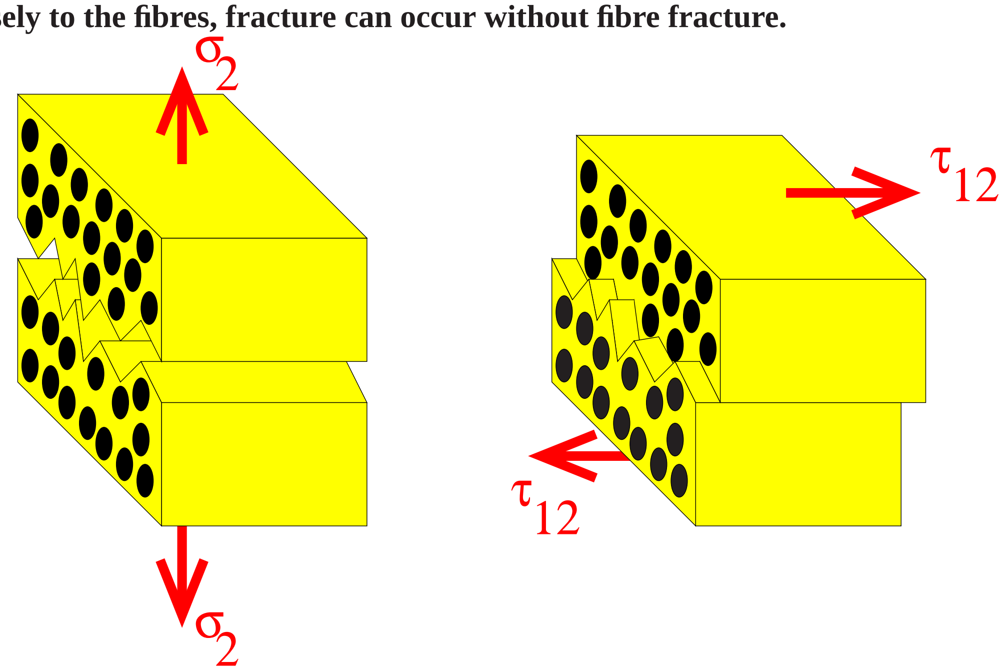
$$\sigma_1^* = \phi_f \sigma_f^* \qquad \text{High } \phi_f$$
$$\sigma_1^* = \phi_f \sigma_f' + (1-\phi_f)\sigma_m^* \qquad \text{Low } \phi_f$$
$$\text{Note: } \sigma_f' = \sigma_f(\epsilon_m^*)$$

- Use whichever is higher (heavy dashed line in Figure).
- Note, in this case fibre always strengthens matrix



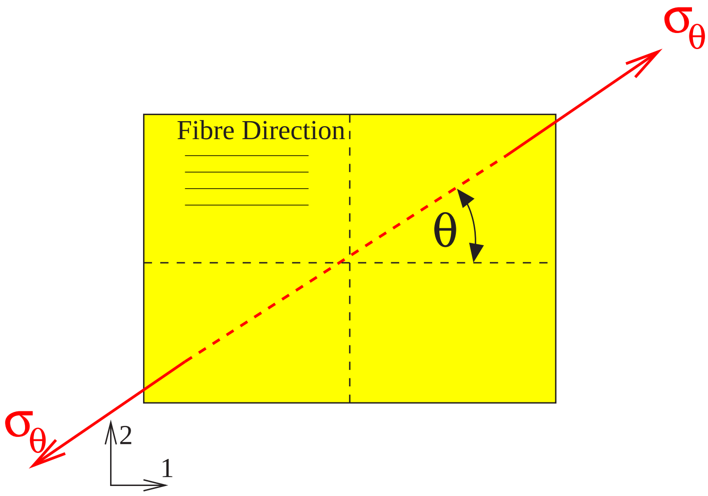
10 Failure of Unidirectional Composite Laminates

When loads are not parallel to the fibres, composite is much weaker. **The high strengths of composites are realied only when the loads are parallel to the fibres. The composite is much weaker under stress in other directions becasue cracks seek out the easiest path along which to popragate. In a Fibre Reinforced Polymer, this will be through the matrix and along the the fibre–matrix interface. When a tensile stress acts transversely to the fibres, fracture can occur without fibre fracture.**



11 Failure of Unidirectional Composite Laminates

11.1 Loading at angle θ to fibres



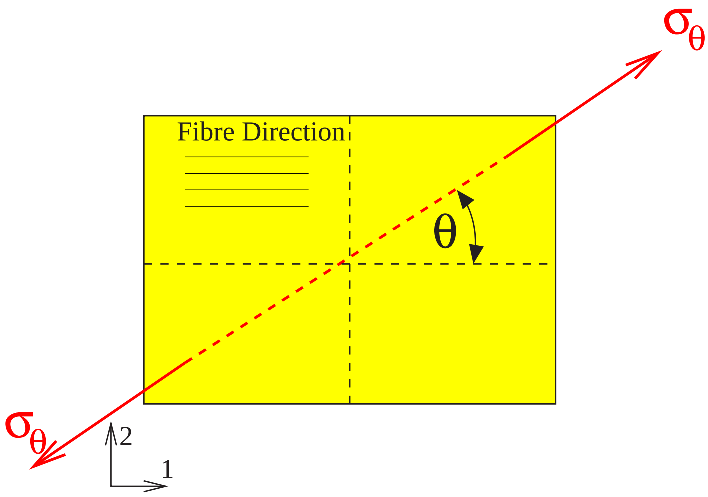
In fact, the fibres can act as stress concentrators, so composite is somewhat weaker than matrix alone. From Mohr's Circle, give σ_θ we have components:

$$\begin{aligned}\sigma_1 &= \sigma_\theta \cos^2\theta \\ \sigma_2 &= \sigma_\theta \sin^2\theta \\ \tau_{12} &= \sigma_\theta \sin\theta\cos\theta\end{aligned}$$

Failure occurs when any one of these three reaches its limiting value

12 Failure of Unidirectional Composite Laminates

12.1 Loading at angle θ to fibres

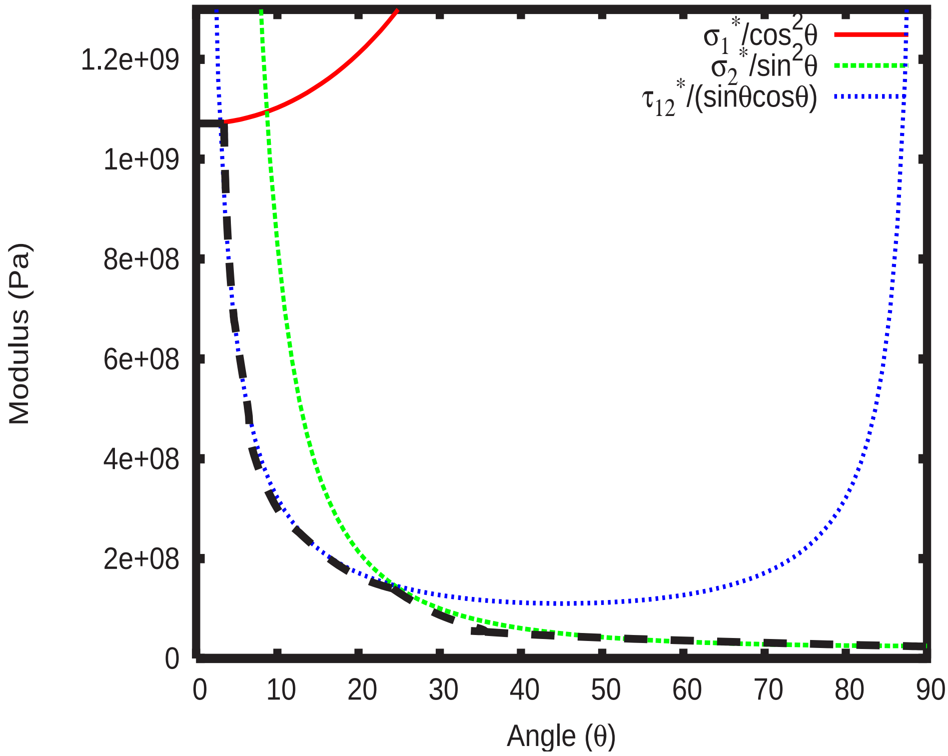


Failure occurs as soon as any of the following occur:

Axial Tensile Failure	$\sigma_\theta^* = \sigma_1^* / \cos^2\theta$
Transverse Tensile Failure	$\sigma_\theta^* = \sigma_2^* / \sin^2\theta$
Axial Shear Failure	$\sigma_\theta^* = \tau_{12}^* / (\sin\theta\cos\theta)$

13 Failure of Unidirectional Composite Laminates

13.1 Loading at angle θ to fibres



Heavy dashed line is plate strength

Note the huge difference in strength between the fibre direction and other directions in the plate. Results based on this theory agree quite well with experiment, as can be seen in Fig. 6.21 of Principles of Polymer Engineering, McCrum and Buckley.

Not only can we predict the stress at which failure will occur, but we can also anticipate the mode of failure.