

6.21 Composite Sheet: 50% by volume Carbon fibres Epoxy matrix. Describe how mode of failure will vary with θ (loading angle)

take $\sigma_2^* = 40 \text{ MPa}$ $\tau_{12}^* = 70 \text{ MPa}$

$\sigma_f^* = 3.2 \text{ GPa}$ $\sigma_m^* = 0.060 \text{ GPa}$

$E_f = 230 \text{ GPa}$ $E_m = 2.4 \text{ GPa}$



$$E_f^* = \frac{\sigma_f^*}{E} = \frac{3.2}{230}$$

$$E_m^* = \frac{\sigma_m^*}{E} = \frac{60}{2400}$$

$$= 0.01391$$

$$= 0.025$$

↑ smaller \Rightarrow fibres brittle

$$\sigma_m' = E_m E_f^* = (2400)(0.01391) = 33.38 \text{ MPa}$$

$$\begin{aligned} \text{Stress } \sigma_1 &= \phi_f \sigma_f^* + \phi_m \sigma_m' \\ &= (0.5)(3200) + (0.5)(33.38) \\ &= 1617 \text{ MPa} \end{aligned}$$

matrix alone $(1 - \phi_f)(\sigma_m^*) = (0.5)(60) = 30 \text{ MPa}$

$30 \ll 1617 \therefore \sigma_1^* = 1617 \text{ MPa}$

small θ $\sigma_0^* = \frac{1617}{\cos^2 \theta}$ failure along the direction

transition Shear // fibres $\frac{\sigma_1^*}{\cos^2 \theta} = \frac{\tau_{12}^*}{\sin \theta \cos \theta} = \sigma_0^*$

$$\tan \theta = \frac{\tau_{12}^*}{\sigma_1^*} = \frac{70}{1617} = 0.04329; \theta = 2.48^\circ$$